

# Maintenance Dependence Modeling with Gaussian Copulas

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## Abstract

The main objective of the paper is to study the response variable -joint cumulative distribution function (CDF) - of two controlled variables, mileage (measured in kilometers) and maintenance costs based on which we predict or explain the output variable. The paper proposes a method based on the bivariate cumulative normal distribution with the Nataf model and compares it to the method of transforming the dependent into independent variables with similar results. The methodology is applied to real-life data assessing the dependence between the traffic urban mileage and the total maintenance costs. Based on copula functions, the Nataf transformation is used to handle the dependence of correlated predictor variables and marginal distributions. Copula requires only marginal CDFs and correlation parameters in order to approximate the joint outcome variable. The main idea of the second applied method is to transform the dependent normal random variables into independent standard ones. The two sets of values of the joint predicted variable were compared with suitable outcome. The proposed statistical models have general application and could be used for technical and economical prognoses and schedules.

**Keywords:** Model, Maintenance, Maintenance costs, Cumulative distribution function, Normal distribution, Nataf Model, Copula functions, Prognose

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